

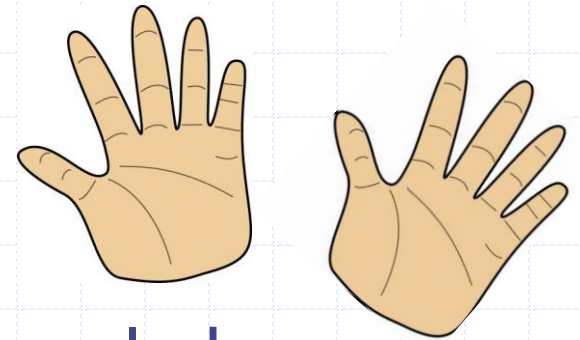
# CESE4030

## Embedded Systems Laboratory

Technology  
Introduction to Control Theory

# Disclaimer / acknowledgements

◆ Koen is a computer scientist



◆ Lecture is based on experts' knowledge



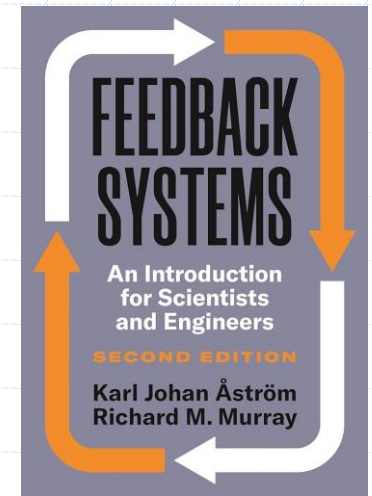
## Introduction to PID Control

Brad Schofield, BE ICS AP



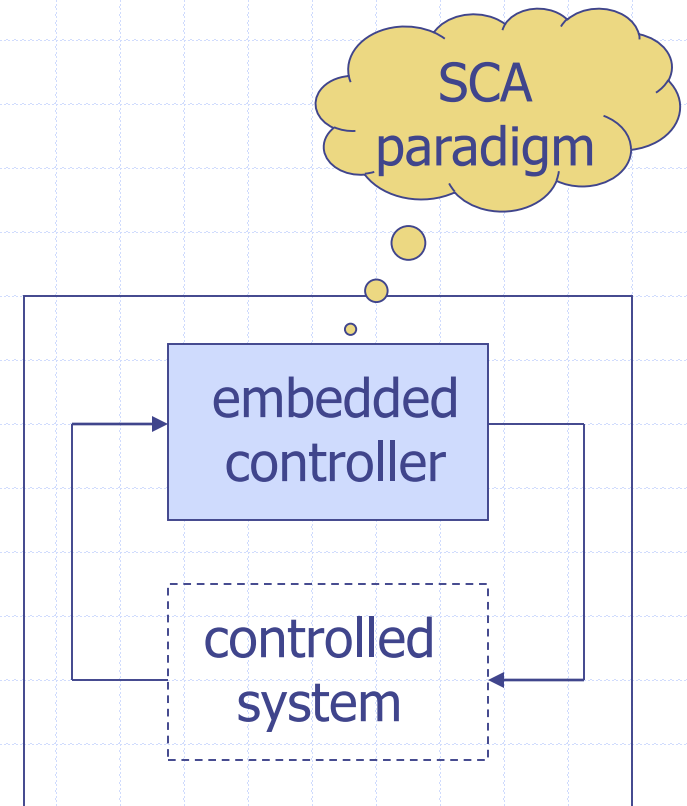
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Brad Schofield

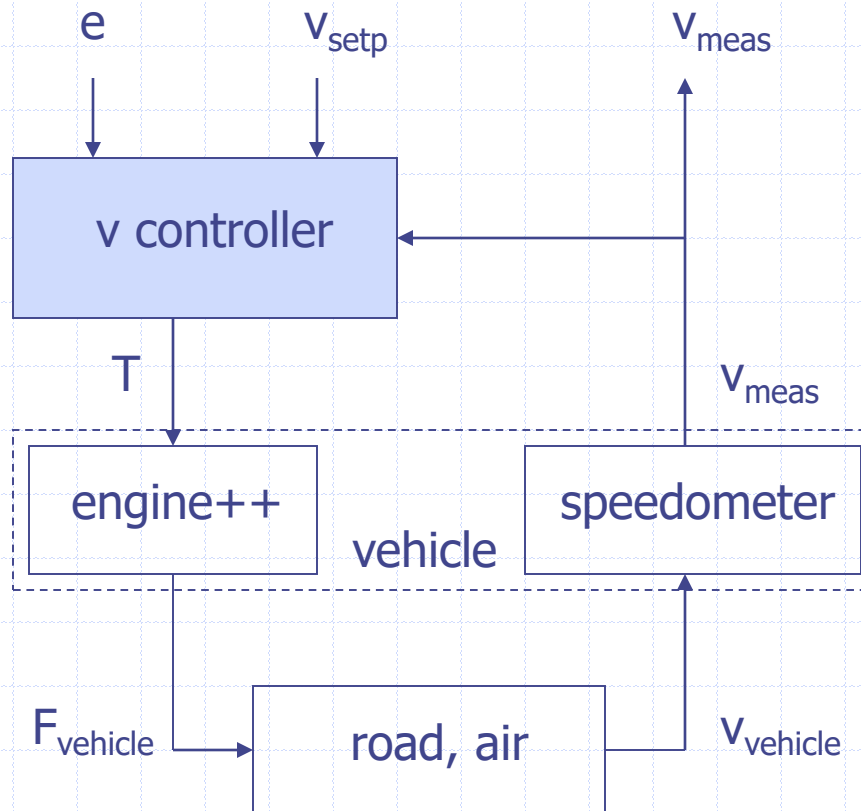


# Control is Everywhere

- ◆ Automotive
- ◆ Aerospace
- ◆ Plant Control
- ◆ Climate Control
- ◆ Health Care
- ◆ Copiers, Wafer Scanners
- ◆ Model Quad Rotors ...



# Cruise Control



$e$  = enable [0/1]  
 $T$  = throttle [%]  
 $F$  = thrust [N]  
 $v$  = velocity [m/s]

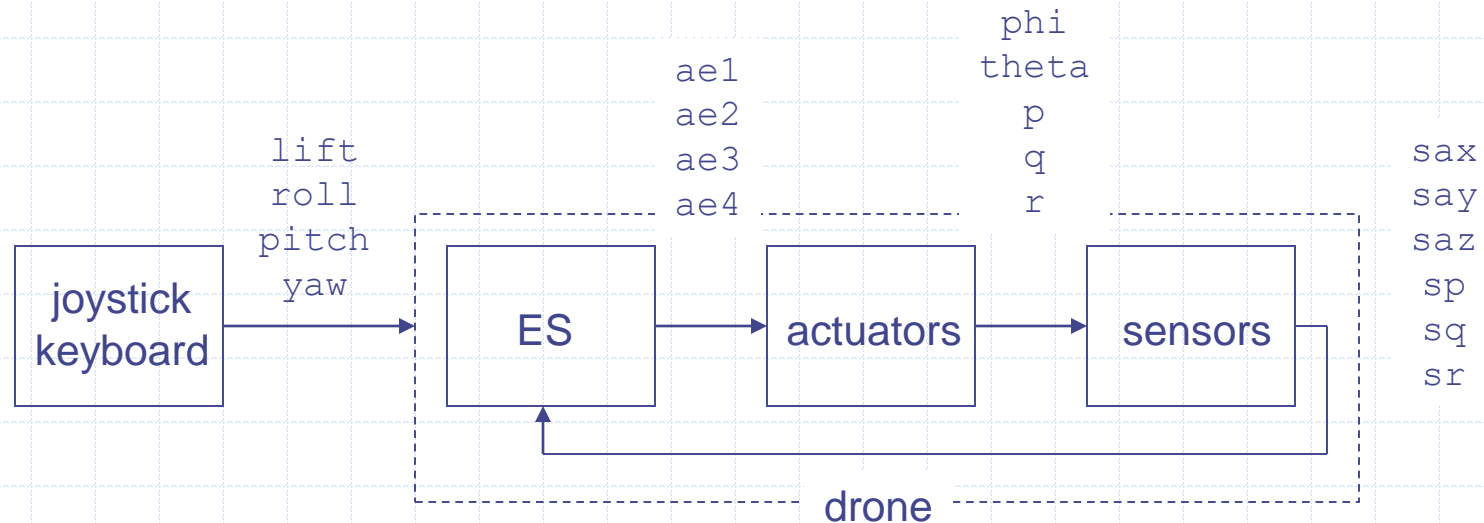
$V_{\text{setp}}$  = setpoint  
 $V_{\text{meas}}$  = measured  
 $V_{\text{vehicle}}$  = actual

disturbances (slope, wind)

# Objectives of this Crash Course

- ◆ Appreciate the benefits of control
- ◆ Understand basic control principles
- ◆ Communicate with control engineers
- ◆ Get you up to speed to do the QR control

# Drone: Control Circuit



control loop example (yaw **rate**):

$\text{eps} = \text{yaw} - \text{sr};$

$N_{\text{needed}} = P * \text{eps};$

$\text{ae1} \dots \text{ae4} = f(N_{\text{needed}});$

// measure deviation

// compute ctl action

// actuate, see slide 9

# Introduction to PID Control

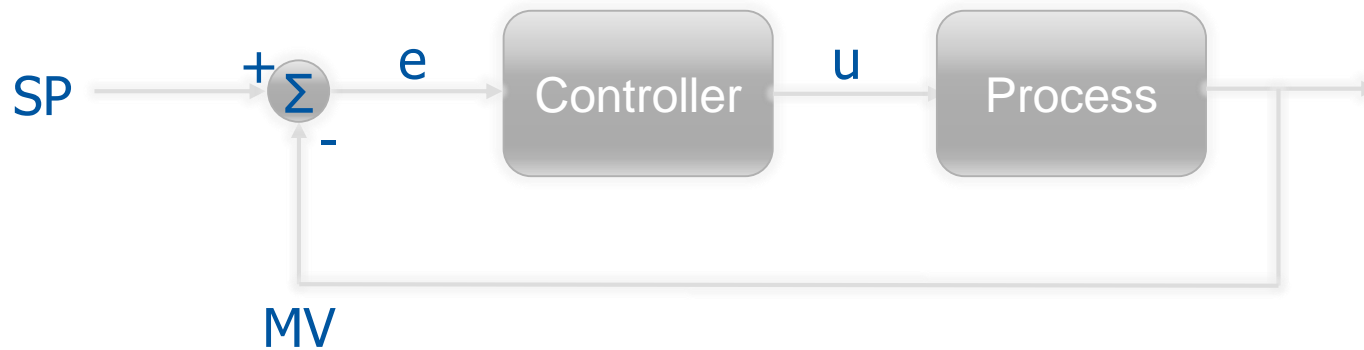
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# What is PID Control?

- Let's take a step back... What is **control**?
- **Control** is just making a dynamic process behave in the way we want
- We need 3 things to do this:
  - A way to **influence** the process
  - A way to see how the process **behaves**
  - A way to **define** how we want it to behave



# The Closed Loop

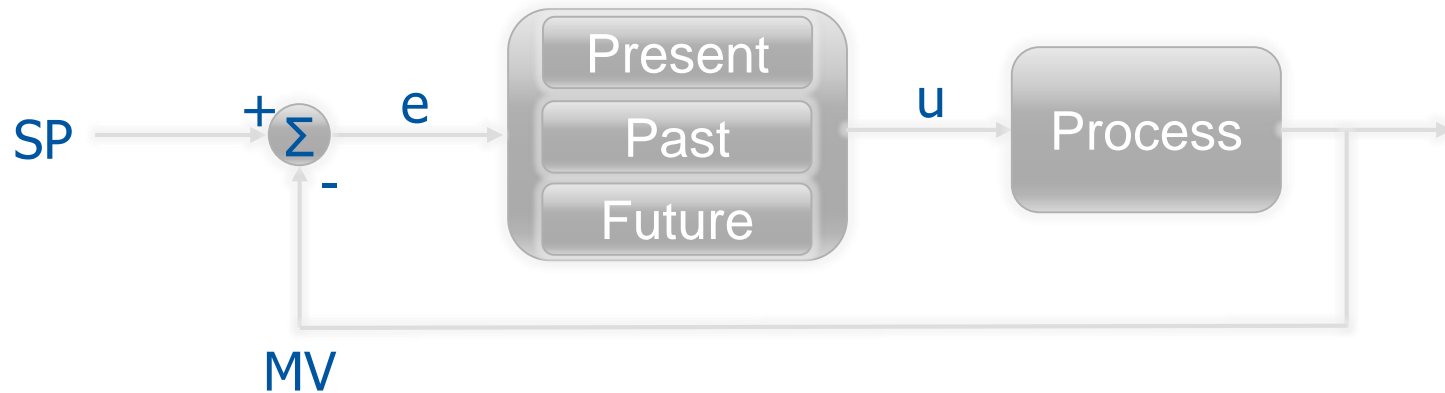


This is the 'classic' closed loop block diagram representation of a control system

# A Dynamic Controller

- We said that since the process is dynamic (dependent on inputs made at different times), it makes sense that the controller should be too
- How do we usually think of time?
  - ‘Present’
  - ‘Past’
  - ‘Future’

# Splitting the Controller



# The 'Present'

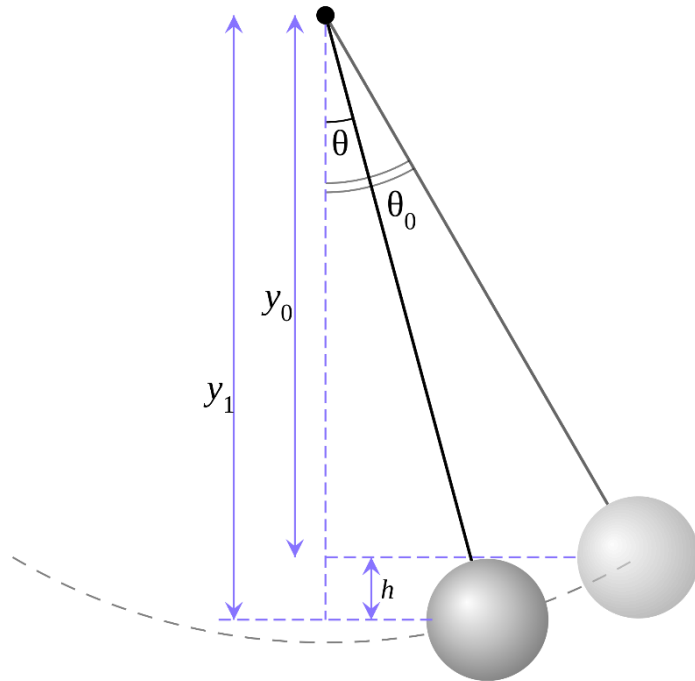
- This part of the controller is only concerned with what the **error** is **now**
- Let's take a simple law: let the control signal be **proportional** to the error:

$$u = k_p \times e$$

# Is Proportional Control enough?

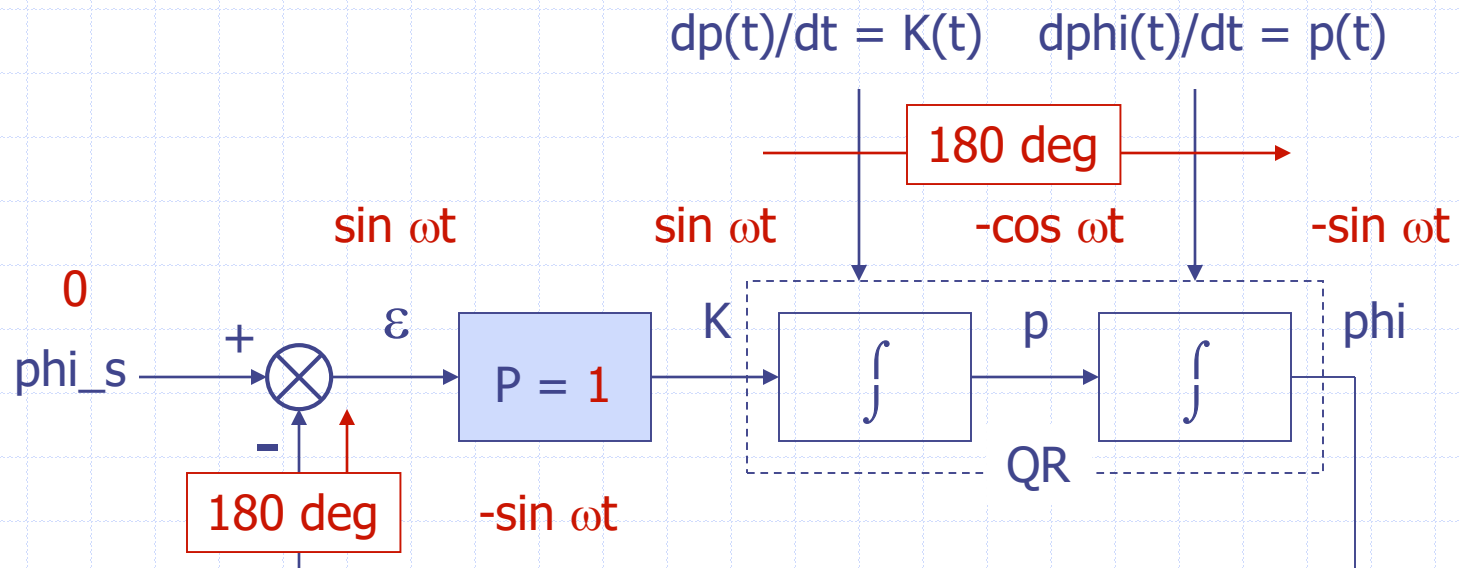
- Intuitively it seems like it should be fine on its own: when the error is big, the control input is big to correct it. As the error reduced so does the control input.
- But there are problems...

# Problem 1: oscillations



Think of a pendulum.  
If the setpoint is hanging straight down, then gravity acts as a proportional controller for the position...  
Pendulum will **oscillate!**

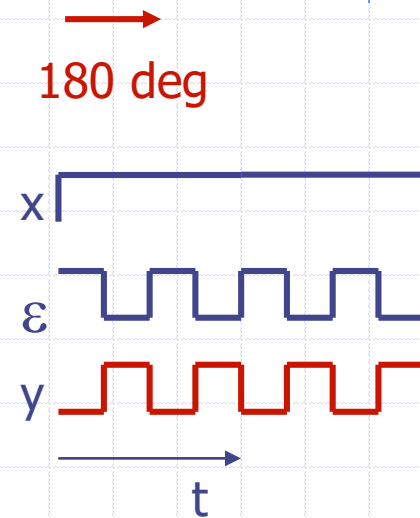
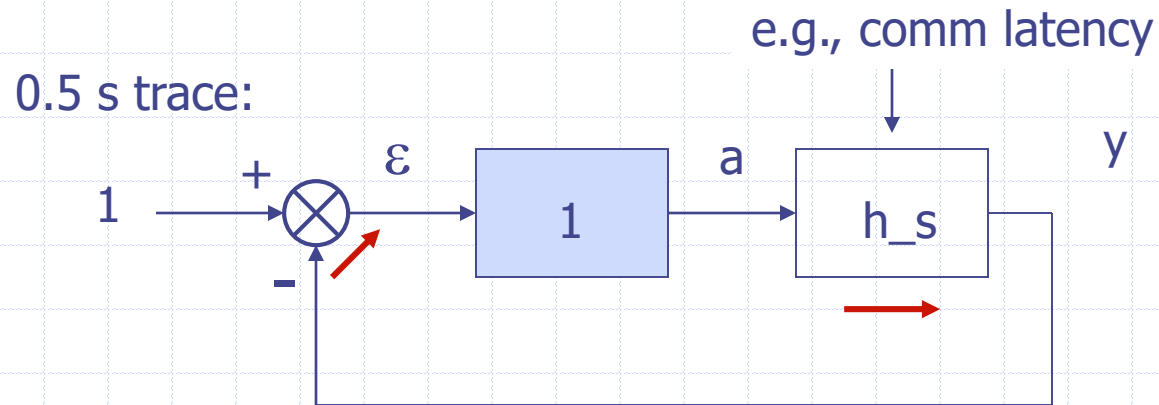
# Example 1: Integrator Systems



$P \geq 1$ : **instability!**

Cause: each integration adds 90 deg phase lag  
So 2 integrators use up all 180 deg budget!

## Example 2: Time Latency



Let  $h_s: y(t) = a(t-0.5)$  (i.e., 0.5s delay)  
Phase lag of 180 deg at 1 Hz causes **instability**



# Phase Lag: examples

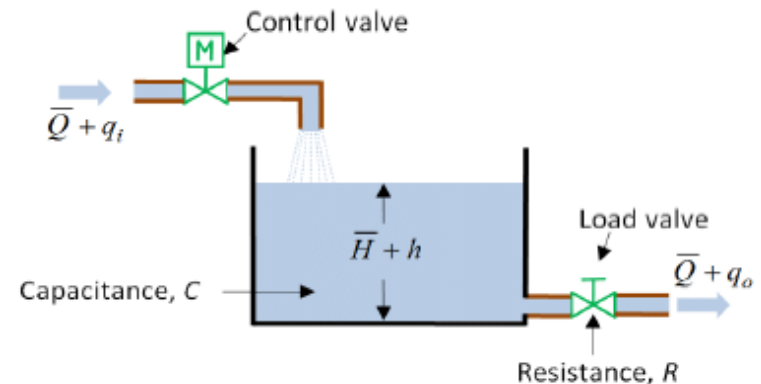
h<sub>s</sub>

- ◆ Integration (90 deg):
  - speed -> position, flow -> volume
- ◆ First-order system (up to 90 deg):
  - lamp, heating, car velocity, ...
- ◆ N-th order system (up to N\*90 deg):
  - compositions of 1st-order systems, missiles
- ◆ Delay systems (unlimited):
  - humans, computers, sample times, cables, air

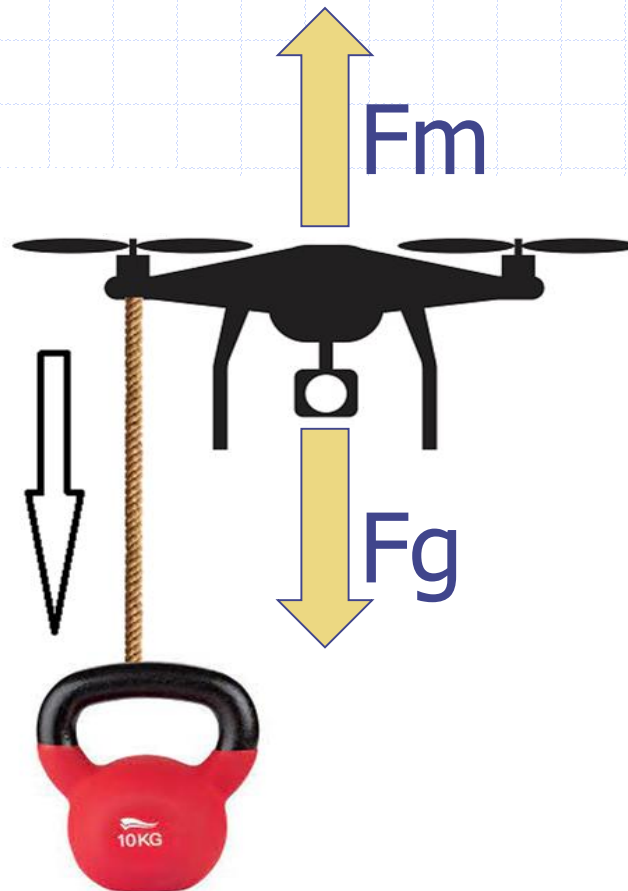
Need control theory to analyze, e.g., control stability

# Problem 2: steady state error

- What happens when the error is zero?
- Causes problems if we need to have a nonzero control value while at our setpoint



# Example: external disturbance





The solution to everything

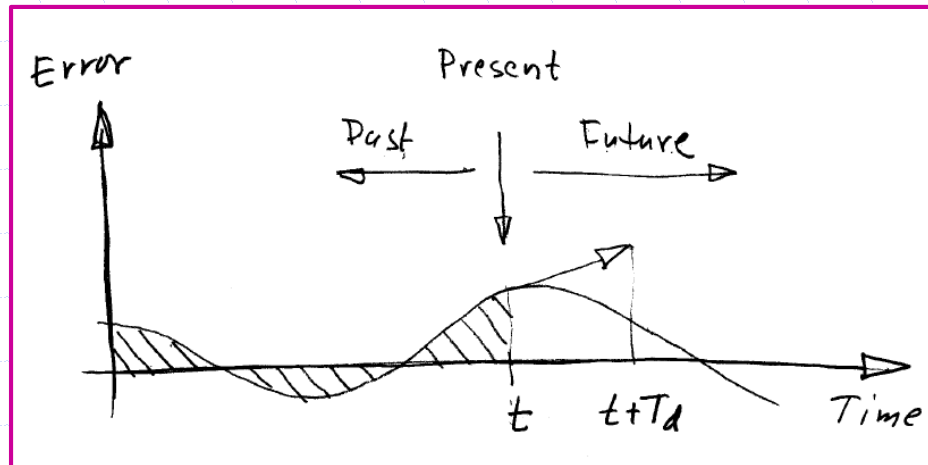
# **PID CONTROL**

# PID in a nutshell

The textbook version of the PID controller is

$$u(t) = ke(t) + k_i \int_0^t e(\tau) d\tau + k_d \frac{de}{dt}$$

$$u(t) = k \left( e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt} \right)$$



# PID control

- ◆ P: address errors
- ◆ D: address oscillations
- ◆ I: address steady state



Who is the boss?

# PID tuning

- ◆ P: address errors
- ◆ D: address oscillations
- ◆ I: address steady state



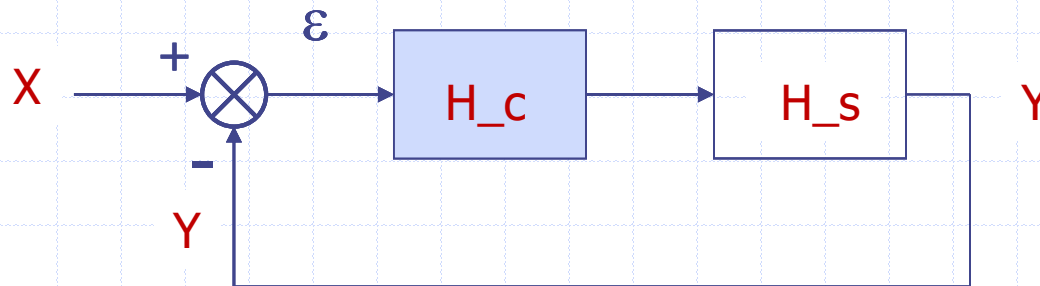
Integral  
windup

Max the  
error



Who is the boss?

# Classical Control Theory

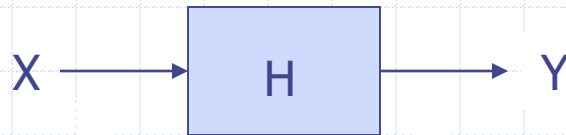


Describe  $x(t)$ ,  $y(t)$ ,  $h_c(t)$ ,  $h_s(t)$  in terms of their Laplace transforms  $X(s)$ ,  $Y(s)$ ,  $H_c(s)$ ,  $H_s(s)$ , respectively

$$L[f(t)] = F(s) = \int_0^{\infty} f(t) e^{-st} dt$$



# Classical Control Theory



For linear system  $h$  it holds  $Y(s) = H(s) \cdot X(s)$  (i.e. composition in time domain reduces to **multiplication** in the Laplace domain). This allows for easy analysis.

# Laplace cheat sheet

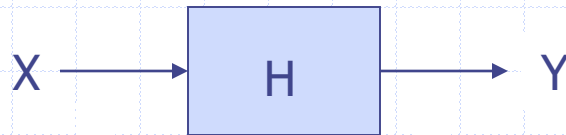
$$\blacklozenge L[a] = a/s$$

$$\blacklozenge L[a t] = a / s^2$$

$$\begin{aligned}\blacklozenge L[a f + b g] &= a L[f] + b L[g] \\ &= a F(s) + b G(s)\end{aligned}$$

$$\begin{aligned}\blacklozenge L[f'] &= s L[f] - f(0) \\ &= s F(s) - f(0)\end{aligned}$$

# Control System Analysis



$$Y(s) = H(s) X(s)$$

$$H(s) = \frac{Y(s)}{X(s)}$$

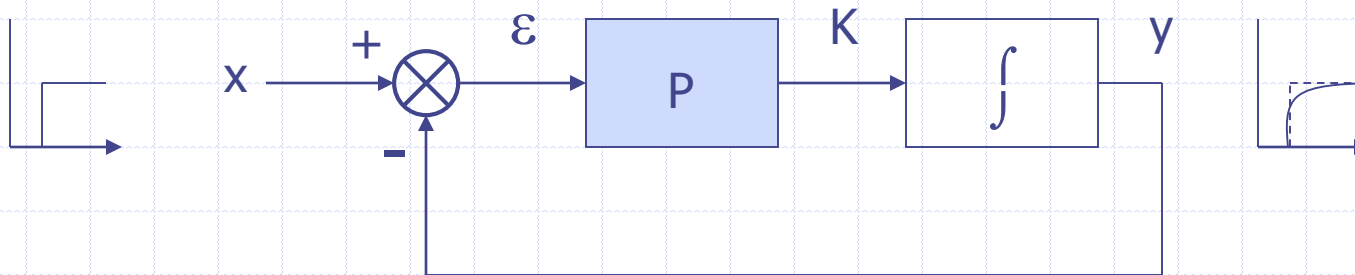
zeros

poles

Stability:

$$\begin{aligned} \operatorname{Re}(\text{roots } H(s)) &< 0 \\ \operatorname{Im}(\text{roots } H(s)) &\text{ small} \end{aligned}$$

# Example: Rate Control



$$Y(s) = P H(s) (X(s) - Y(s))$$

$$Y(s) = (P H(s) / (1 + P H(s))) X(s) = H_{PC}(s) X(s)$$

$$H(s) = 1/s$$

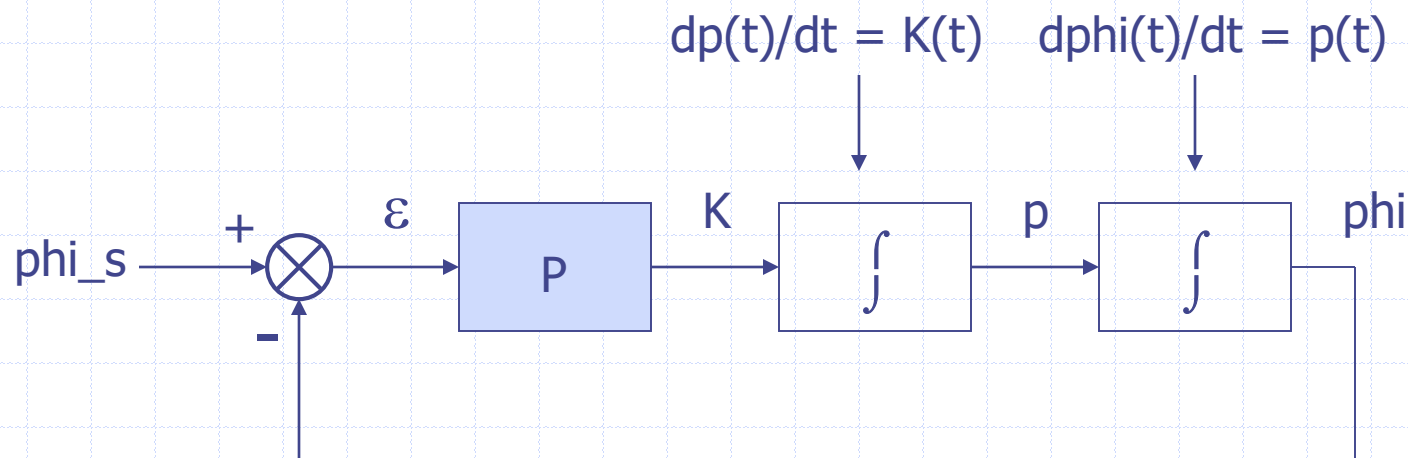
$$H_{PC}(s) = (P/s) / (1 + P/s) = P / (s + P)$$

First-order system with time constant  $1/P$

(root:  $s = -P \Rightarrow \text{Re} < 0, \text{Im} = 0$ ) so **stable**

# Angle control using P controller

P controller for roll angle:

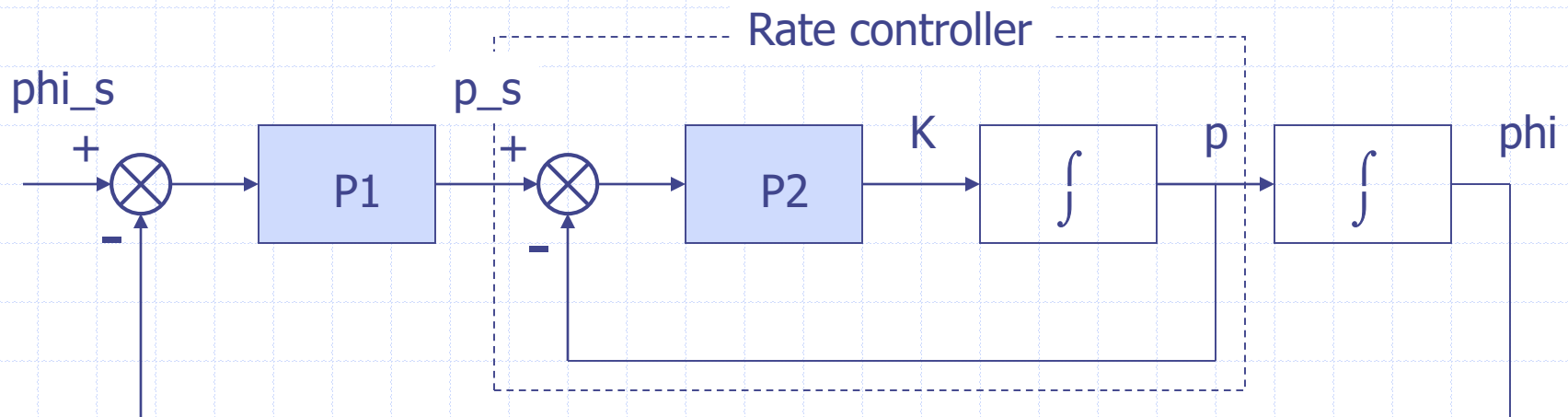


$P < 1$ : useless control performance

$P \geq 1$ : **instability**

# Angle control using cascaded P control

Embedded rate controller “neutralizes” one integrator



Cascaded P Controller: **stable** (for not too high  $P_1$  and  $P_2$ !  
and  $P_2 \gg P_1$ )

# Summary

- ◆ Feedback control offers many advantages
- ◆ Is ubiquitous (cars, planes, missiles, QRs ..)
- ◆ Potential stability problems
- ◆ Need control theory
- ◆ This was merely introduction into the field
- ◆ Get a feel by applying to QR!