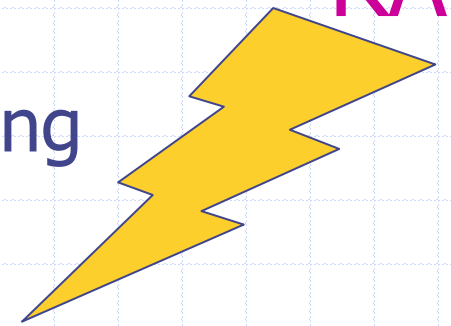
CESE4030
Embedded Systems Laboratory

System Integration

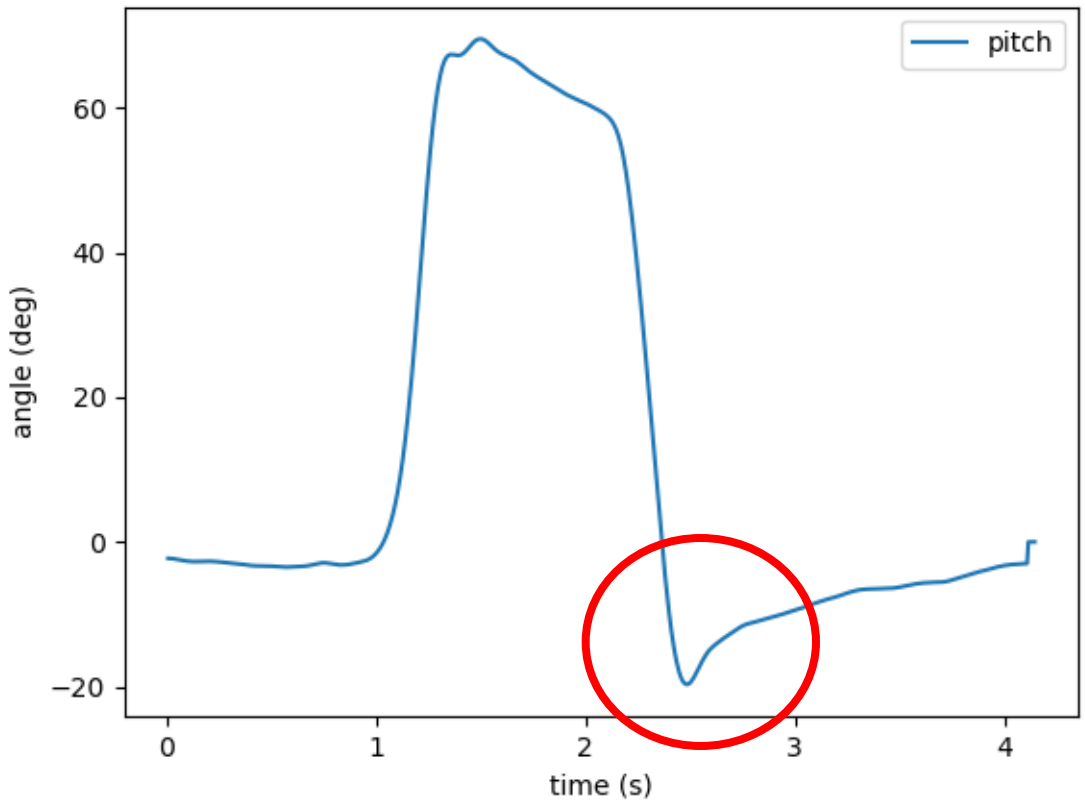
takes time

Putting it all together

1. System identification
2. Embedded programming
3. Control theory
4. Signal processing

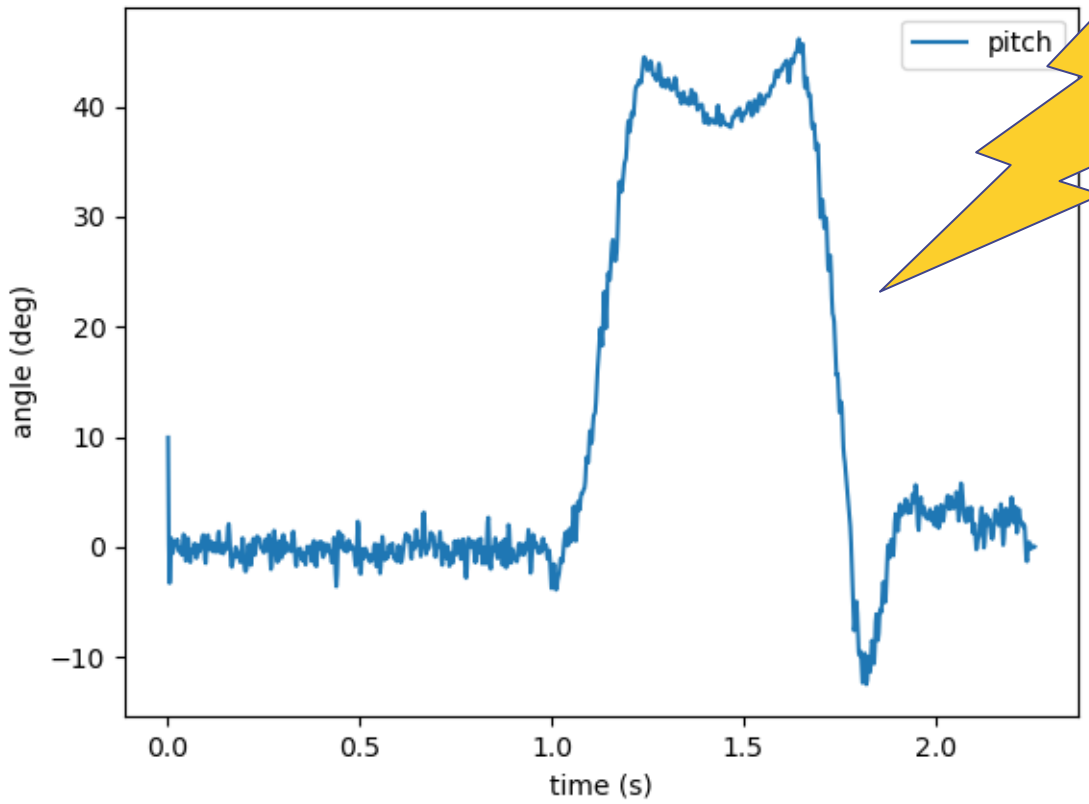


Signals – DMP



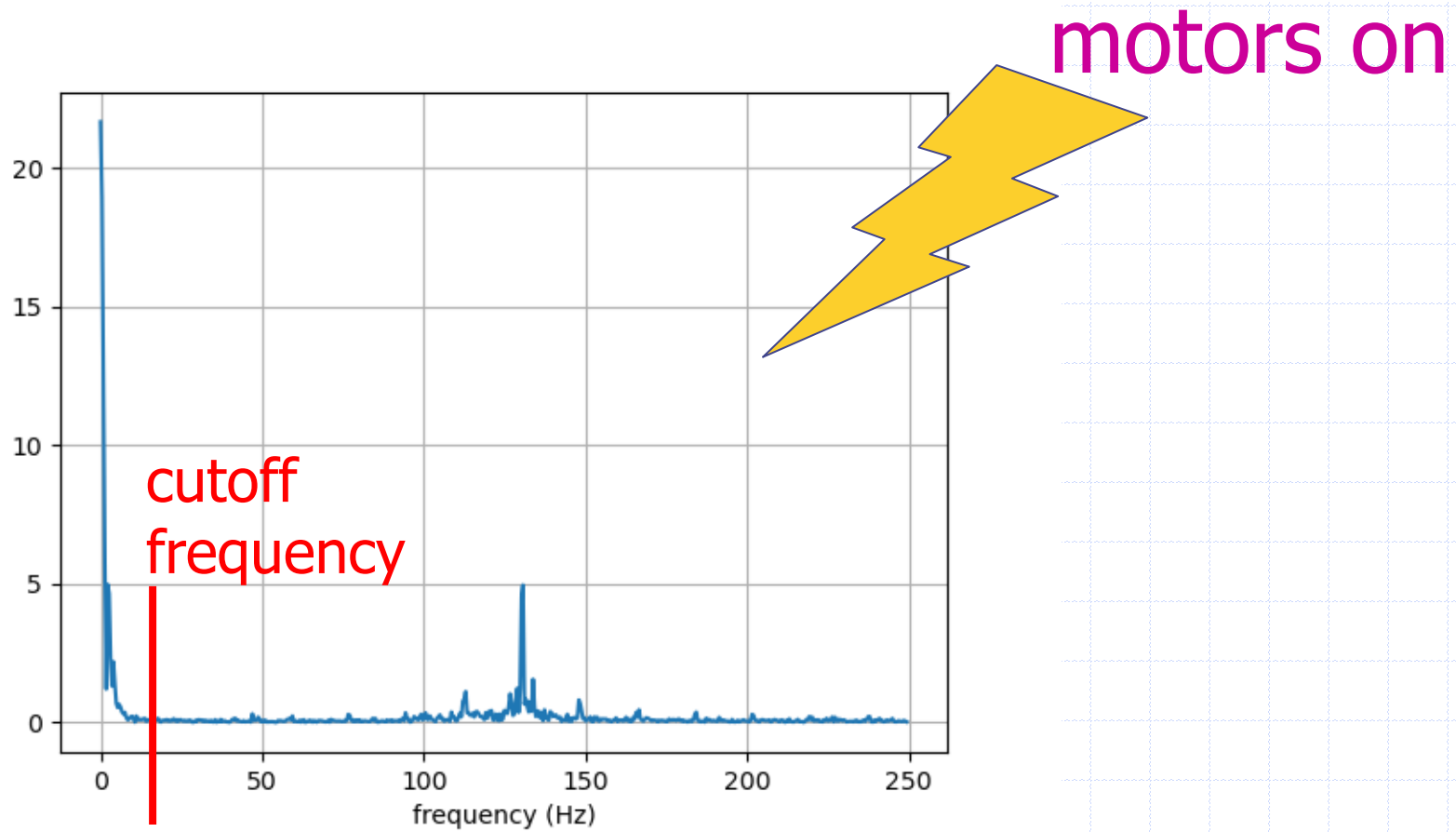
Signals – accelerometer

motors on

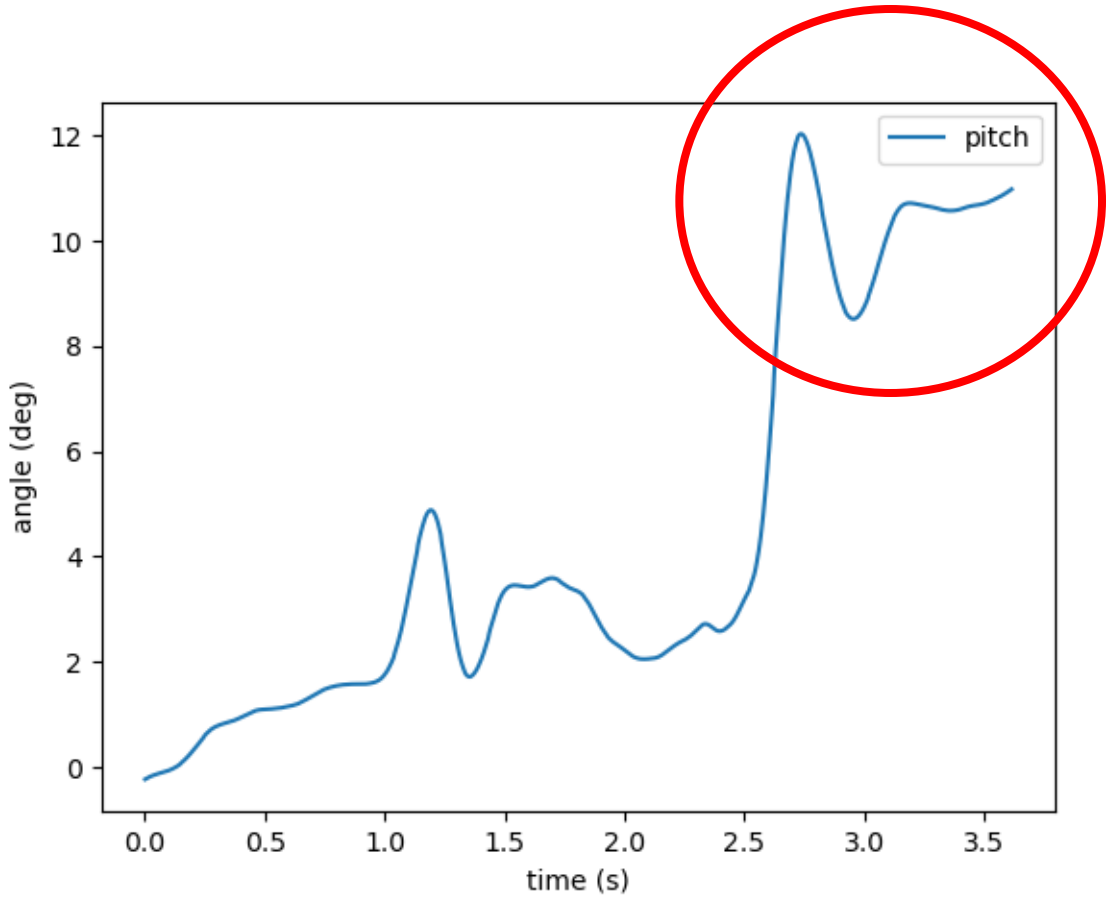


$$\theta = \arctan\left(\frac{sax}{saz}\right)$$

FFT – accelerometer

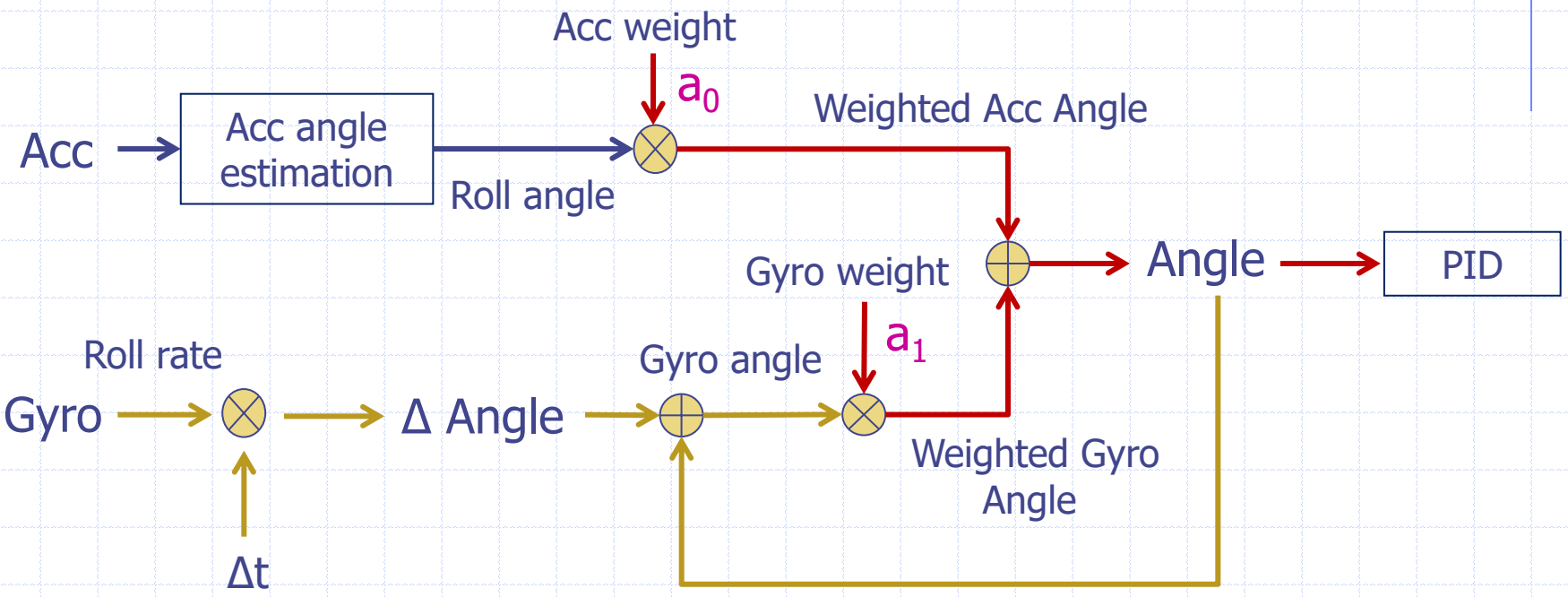


Signals – gyro



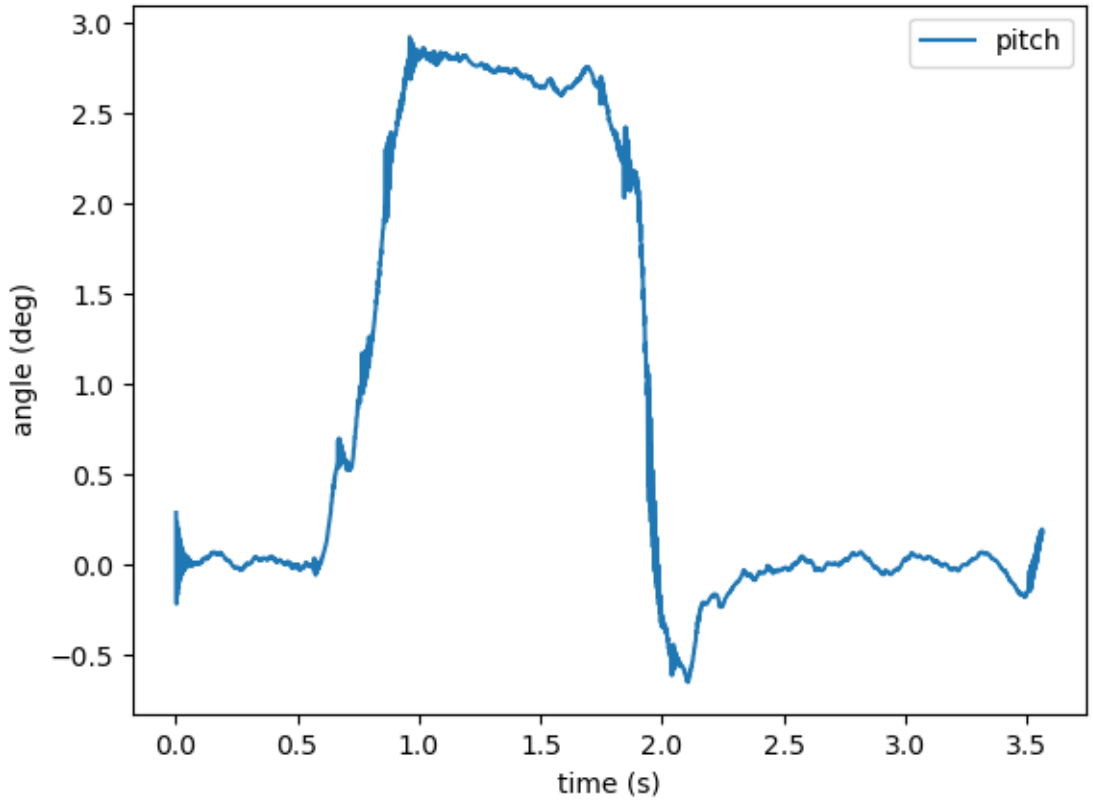
$$\theta = \int q$$

Complementary Filter

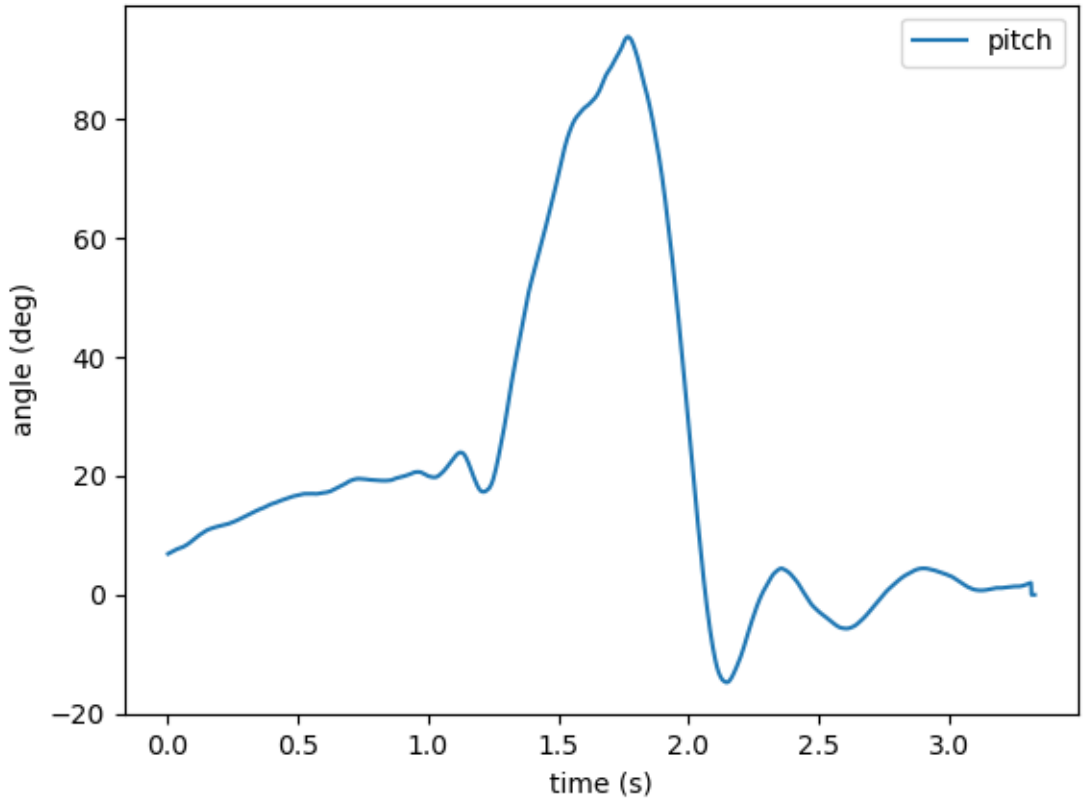


$$theta[n] = a_0 * sax[n] + (1-a_0) * (theta[n-1] + sq[n] * \Delta t)$$

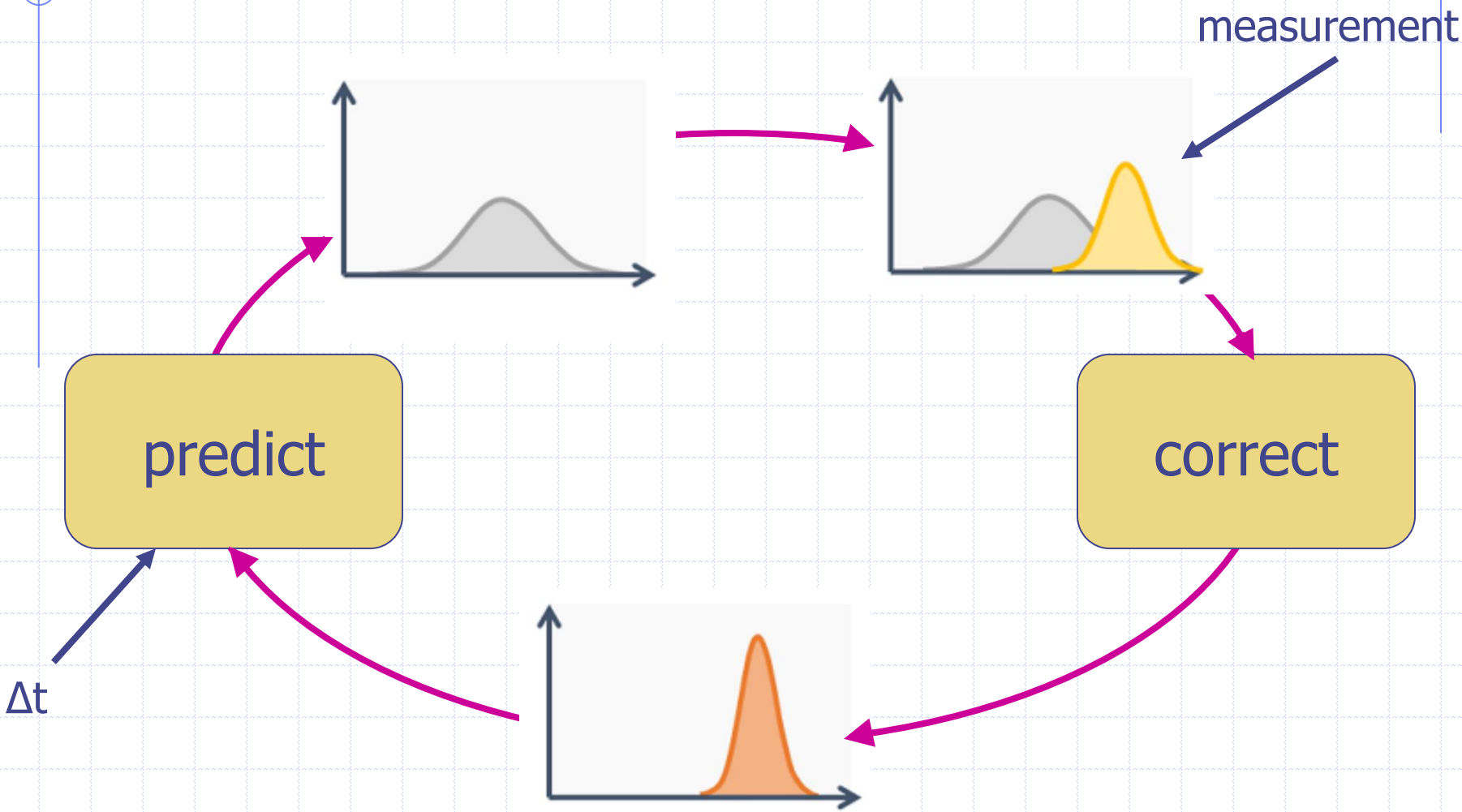
Complementary Filter – 90%



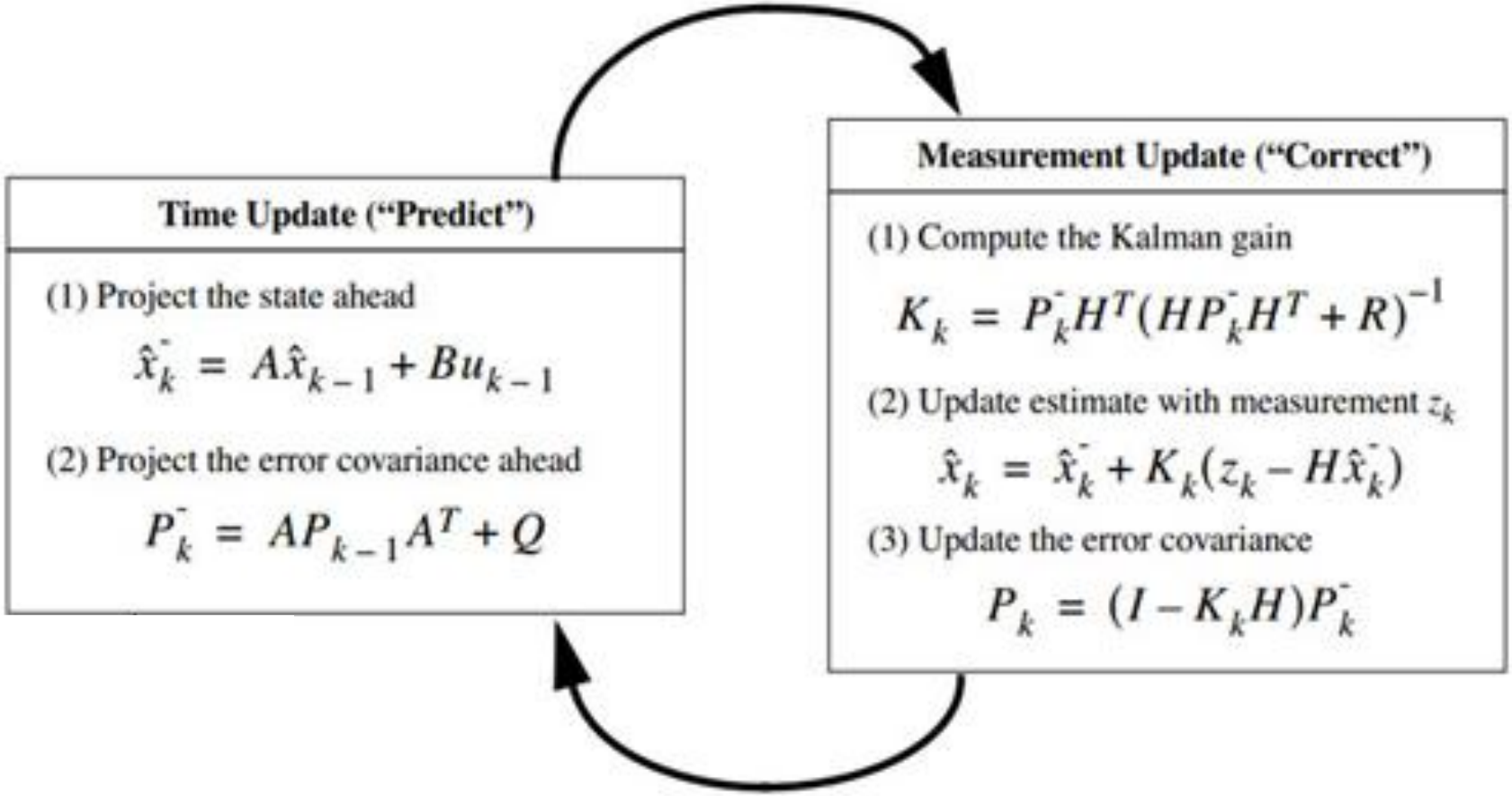
Complementary Filter – 99%



Kalman Filter – sensor fusion

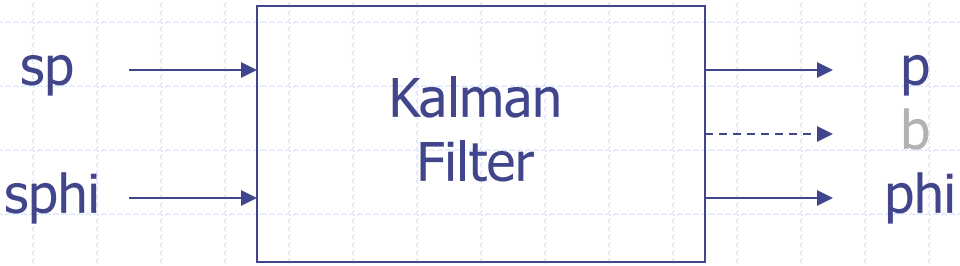


Kalman Filter – sensor fusion



Kalman Filter (simplified)

- Sensor Fusing: gyro and accel share same information



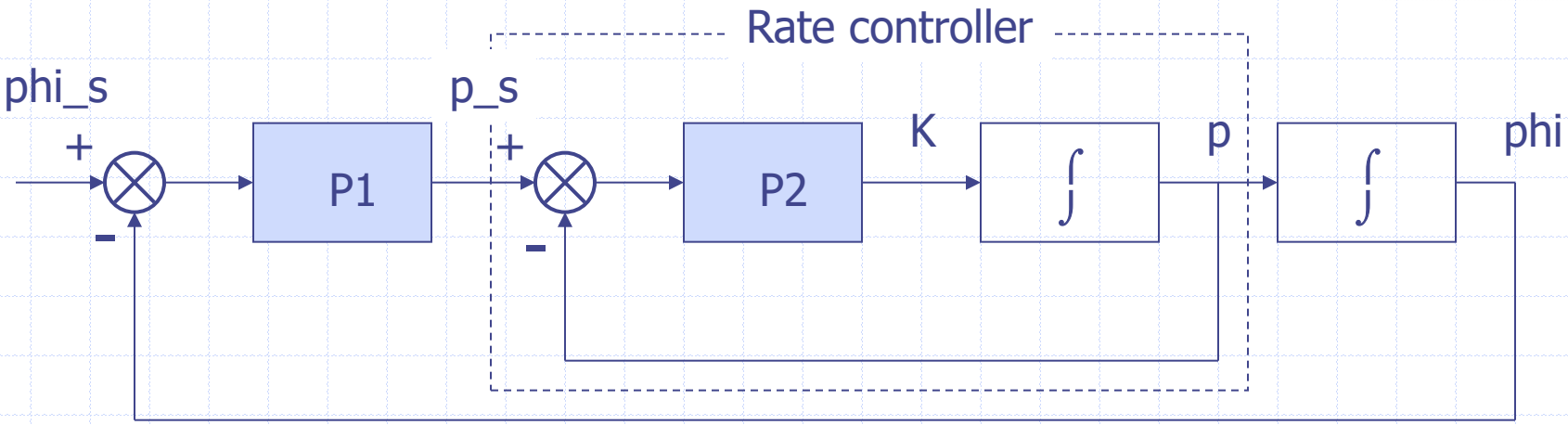
- Integrate sp to phi
- *Adjust* integration for sp (drift) bias *b* by comparing phi to sphi, averaged over *long* period (phi ~ constant)
- Return phi, and p (= sp – bias)

Algorithm

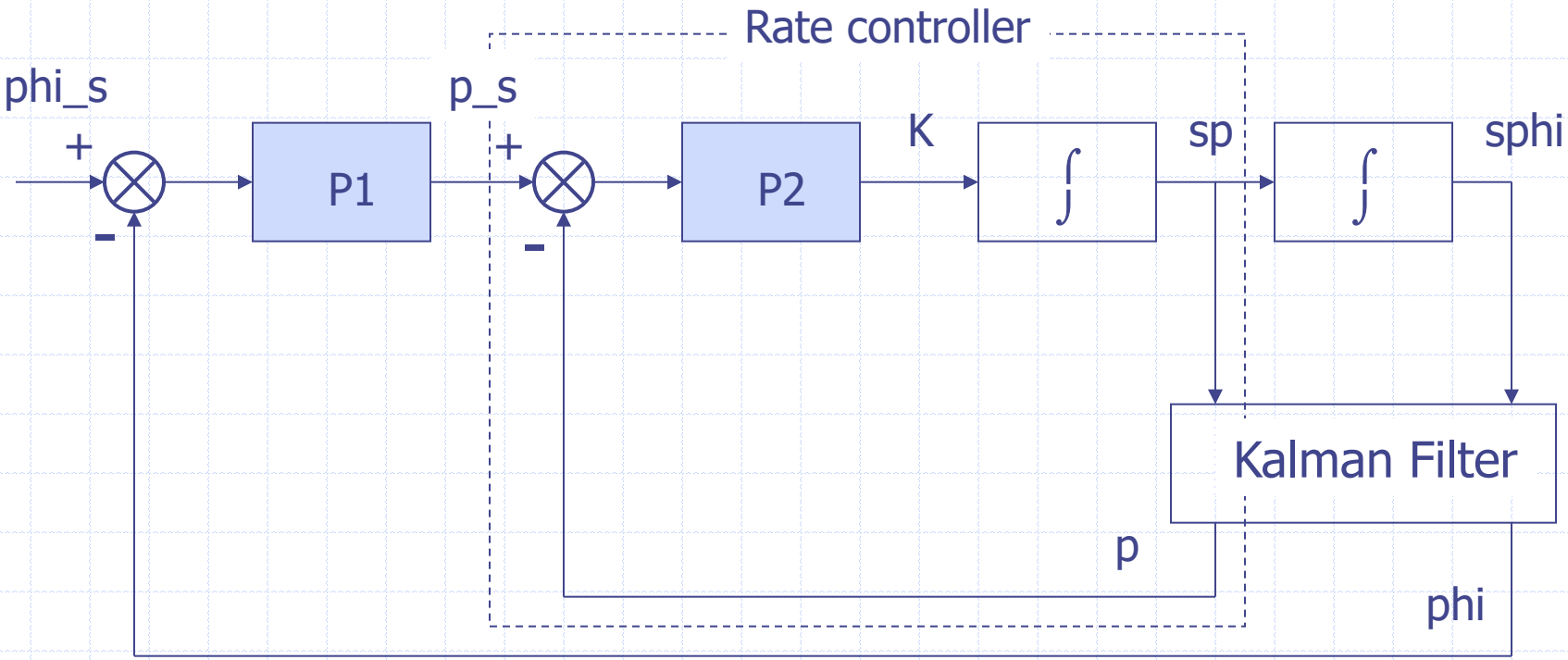
- p = sp – b // estimate real p
- phi = phi + p * P2PHI // predict phi
- e = phi – sphi // compare to measured phi
- phi = phi – e / C1 // correct phi to *some* extent
- b = b + (e/P2PHI) / C2 // adjust bias term

- P2PHI: depends on loop freq -> compute/measure
- C1 small: believe sphi ; C1 large: believe sp
- C2 large (typically > 1,000 C1): slow drift

Cascaded P control



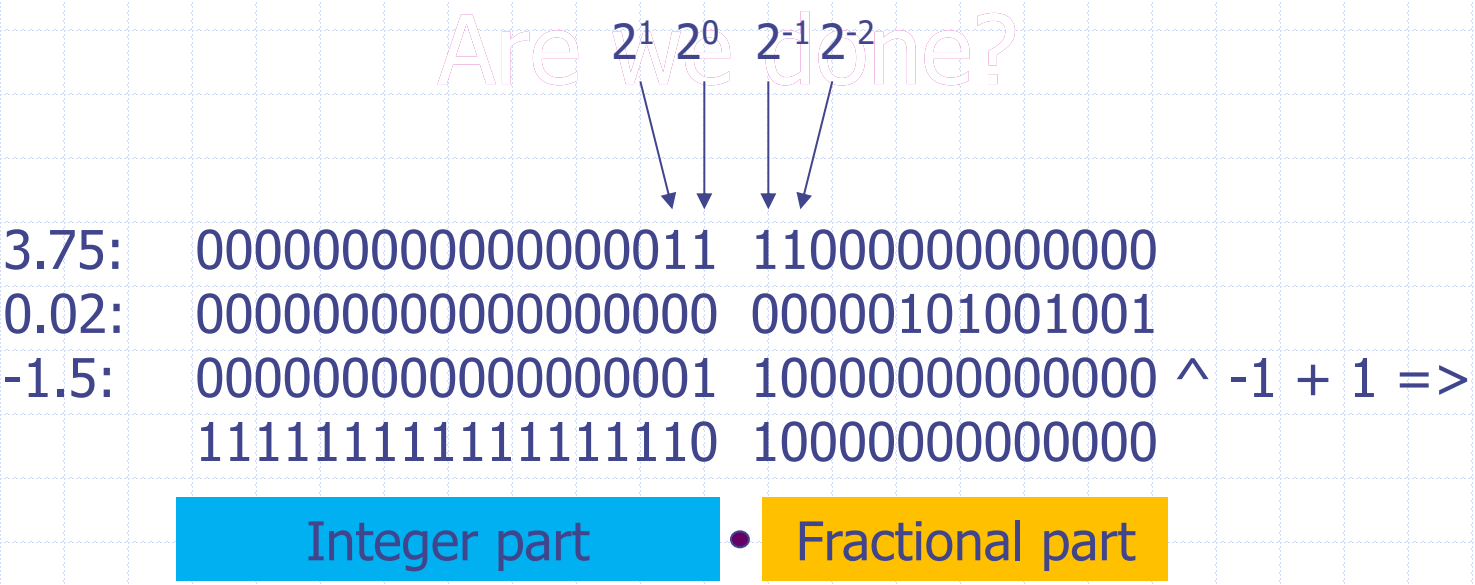
Cascaded P control + Kalman filter



Fixed-point Arithmetic

- Simple/cheap microcontrollers have no floating-point unit
- Software floating-point often (too) slow
- Need to implement filters in fixed-point arithmetic

2's-complement bit representation (e.g., 32 bits, 14 bits fraction):



Fixed-point Arithmetic

- Addition, subtraction as usual
- Multiplication: result must be post-processed:
  - make sure intermediate fits in variable! (e.g., 32 bits)
  - shift right by |fraction|

Example multiplication (32 bits, 14 bits fraction):

3.75: 00000000000000011110000000000000 times:
-1.5: 11111111111111101000000000000000 equals:
       10100110000000000000000000000000
       (value just fits in 32 bits!)
       (now shift right by 14 bits and sign-extend):
       11111111111111010011000000000000 which is:
-5.625 1111111111111010 01100000000000

Filter Example

- Second-order Butterworth LP Filter $f_c = 10Hz$, $f_s = 1250Hz$

$$y[n] = a_0\, x[n] + a_1\, x[n-1] + a_2\, x[n-2] - b_1\, y[n-1] - b_2\, y[n-2]$$

- Coefficients:

$a_0 = 0.0006098548$ $a_1 = 2\, a_0$ $a_2 = a_0$

$b_0 = 1$ $b_1 = -1.9289423$ $b_2 = 0.9313817$

Bit representation (e.g., 32 bits, 14 bits fraction):

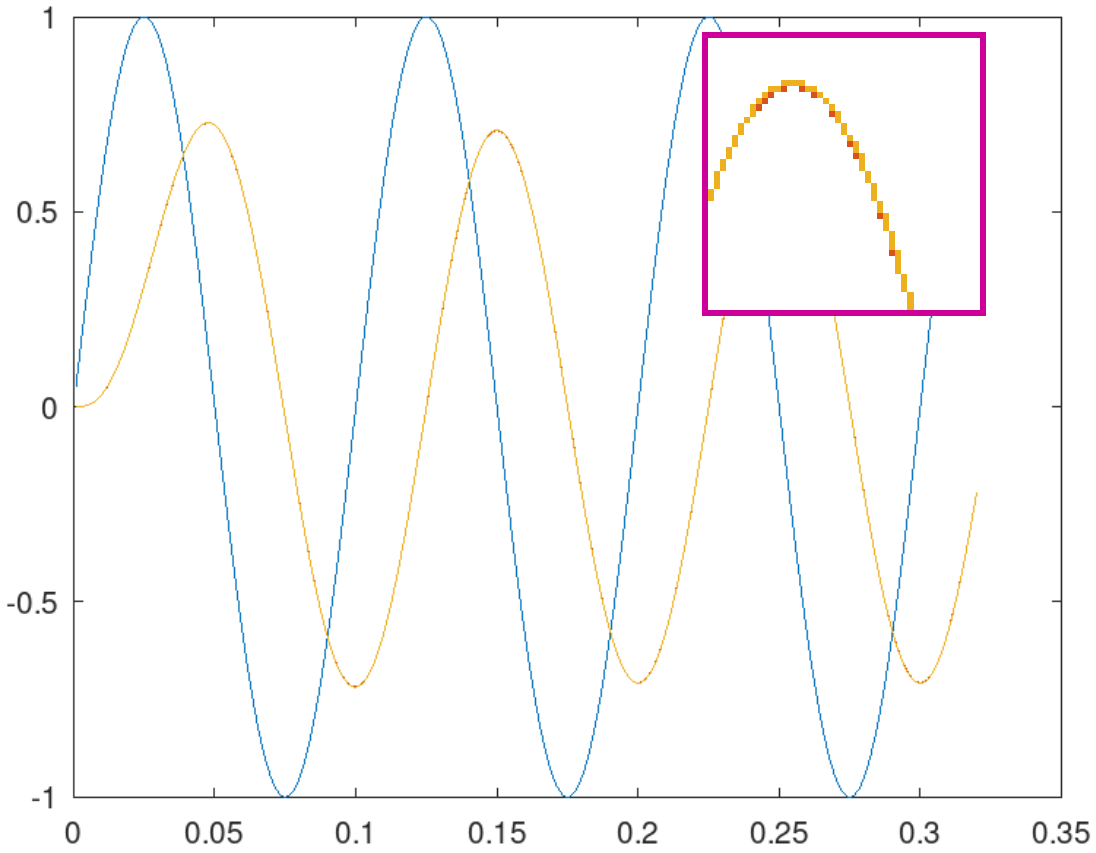
a[0]	00000000000000000	00000000001010	(a_0 << 14)
a[1]	00000000000000000	00000000010100	
a[2]	00000000000000000	00000000001010	
b[1]	00000000000000001	11101101110100	^ -1 + 1
b[2]	00000000000000000	11101110011100	

```
pkg load signal

% 2nd-order Butterworth - fc 10 Hz at fs 1250 Hz
nmax = 400;
n = (1:nmax);
fs = 1250;
f = 10;
t = n / fs;
x = sin(2*pi*f*n/1250);

% real coefficients
[a,b] = butter(2, f/(fs/2));
a0 = a(1); a1 = a(2); a2 = a(3); b1 = b(2); b2 = b(3);
x1 = 0; x2 = 0; y1 = 0; y2 = 0;
for i = 1:nmax
    x0 = x(i);
    y0 = a0 * x0 + a1 * x1 + a2 * x2 - b1 * y1 - b2 * y2;
    y(i) = y0;
    x2 = x1; x1 = x0; y2 = y1; y1 = y0;
end;
```

Filter Example

```
% 14 bits fraction, approximated in terms of powers of 2
a0 = 8 + 2; % 9.9919
a1 = 2 * a0;
a2 = a0;
b0 = 16384; % 0x1<<14
b1 = -(16384 + 8192 + 4096 + 2048 + 512 + 256 + 64 + 32 + 16 + 4);
b2 = 8192 + 4096 + 2048 + 512 + 256 + 128 + 16 + 8 + 4;

x1 = 0; x2 = 0; y1 = 0; y2 = 0;
for i = 1:nmax
    x0 = x(i);
    y0 = (a0 * x0 + a1 * x1 + a2 * x2 - b1 * y1 - b2 * y2) / 16384;
    z(i) = y0;
    x2 = x1; x1 = x0; y2 = y1; y1 = y0;
end;
```

Results



- blue: input
- red: filtered (real)
- yellow: filtered (FP)

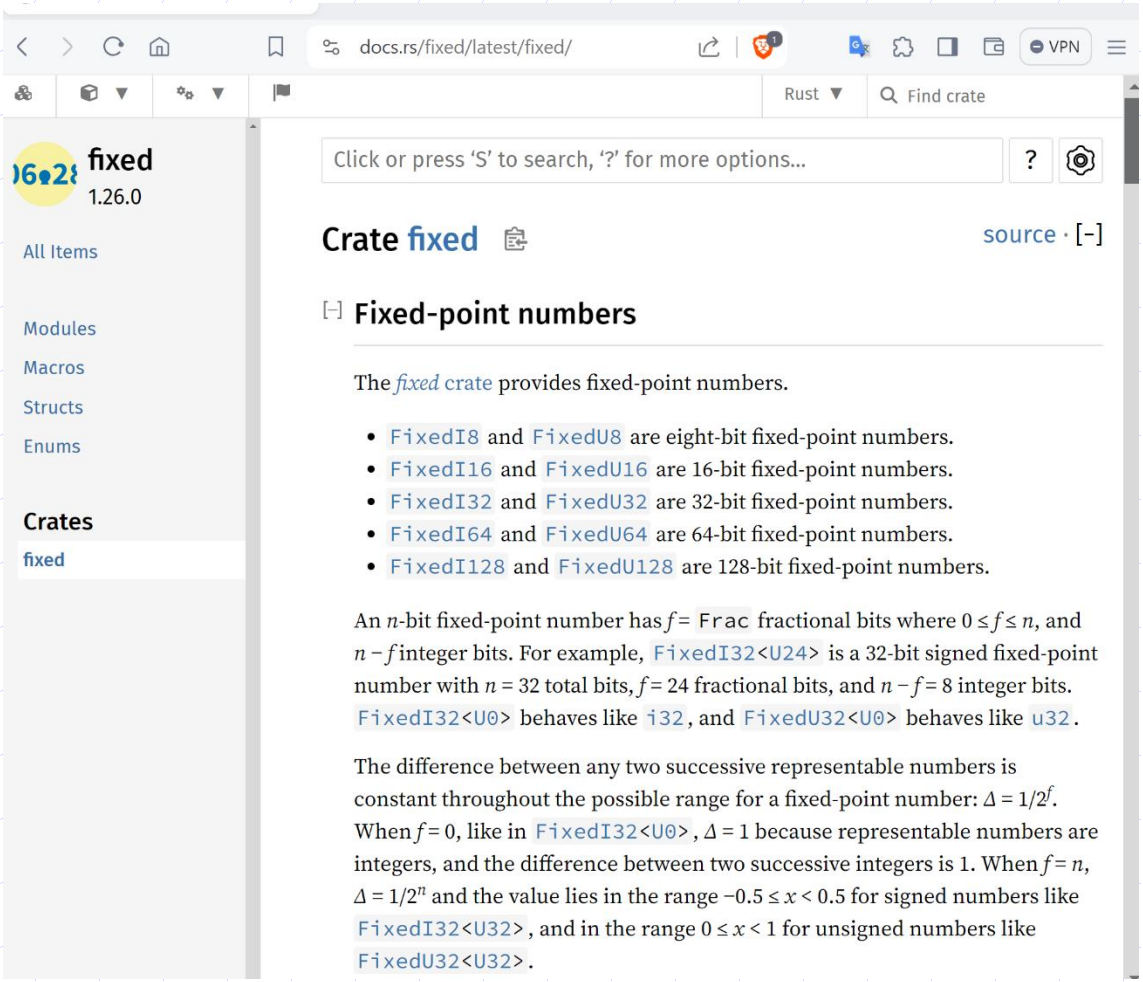
Implementation (high-cost)

```
int mul(int c, int d) {
  int result = c * d;
  return (result >> 14);
}

void filter() {
  y0 = mul(a0,x0) + mul(a1,x1) + mul(a2,x2) -
       mul(b1,y1) - mul(b2,y2);
  x2 = x1; x1 = x0; y2 = y1; y1 = y0;
}
```

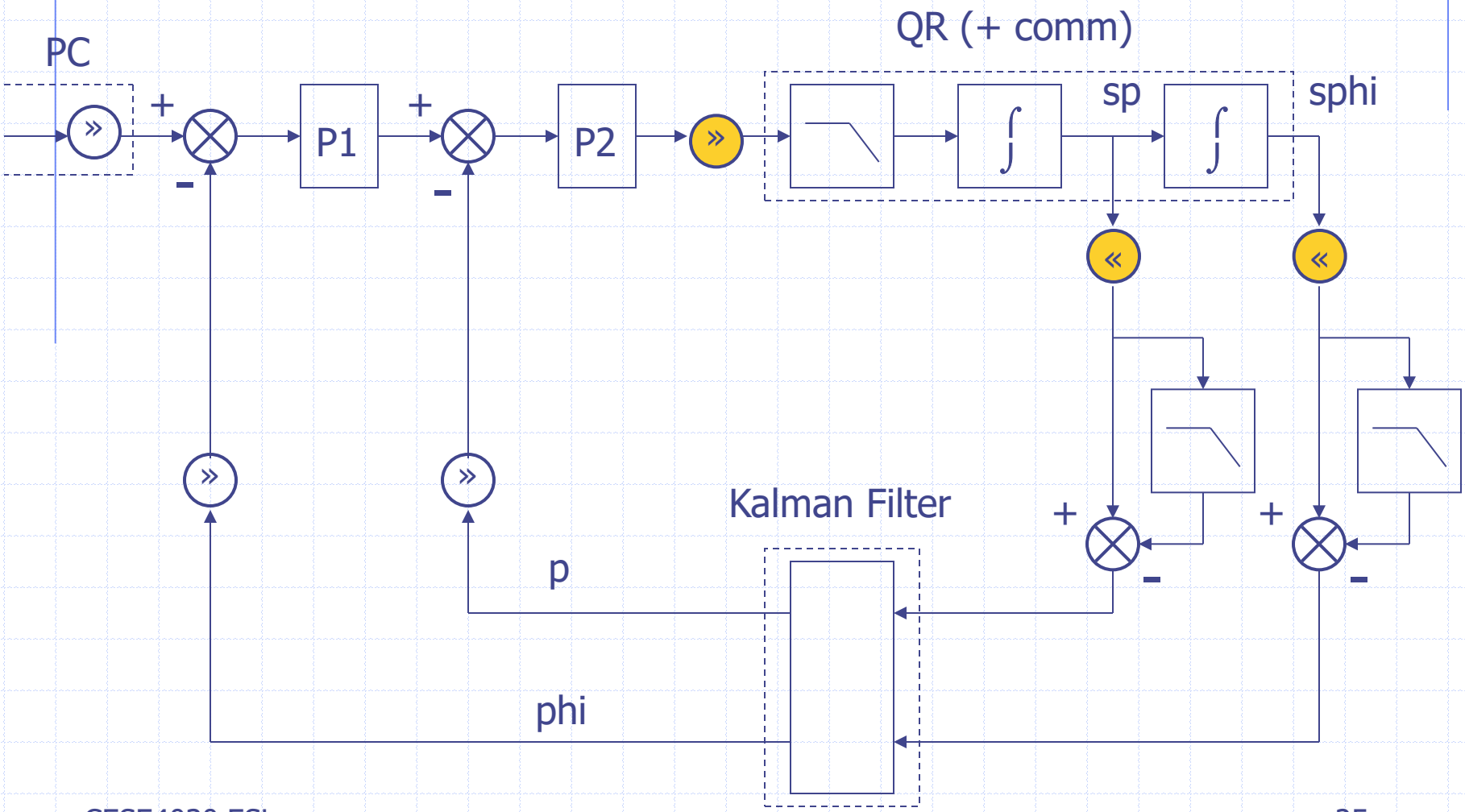
Scaling: tips and tricks

- One size fits all? NO!
  - number of bits depends on needed precision (sensor vs. joystick)
  - special case for proportional controller: P * ε
  - $fp_n * fp_n = fp_{2n}$ (overflow! requires an additional shift)
  - scalar * $fp_n = fp_n$ (overflow? no shift needed)
  - $fp_m * fp_n = fp_{m+n}$ (when P can't be represented as a scalar)
  - document precision for every data type (part of softw arch)
- fp_n to scalar
  - be patient, shift at last instant (when feeding the engines)

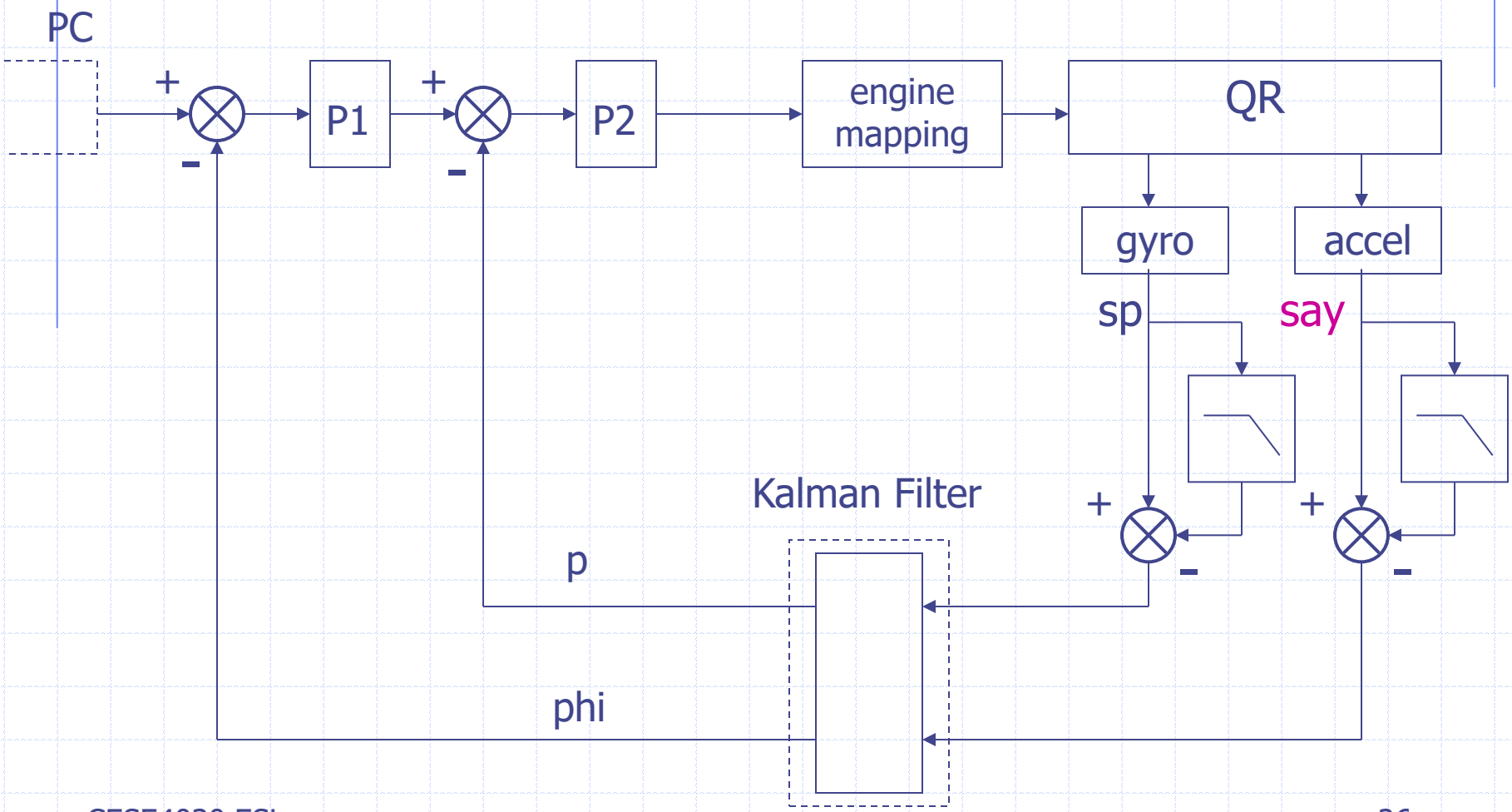
Fixed – the Rust way



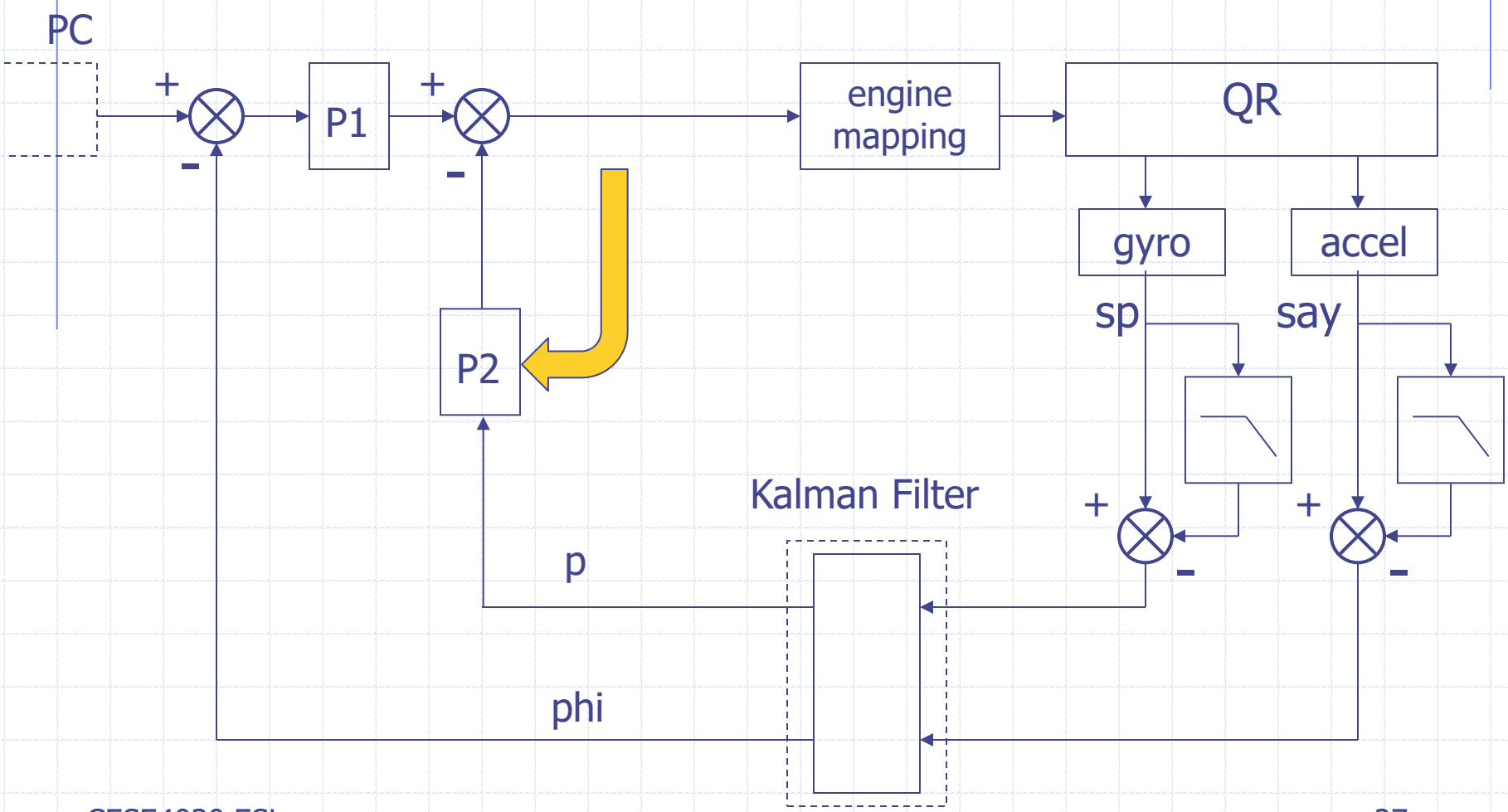
Example Signal Flow (Kalman Filter)



Signal Flow (Kalman Filter)



Signal Flow (Kalman Filter)



PID tuning

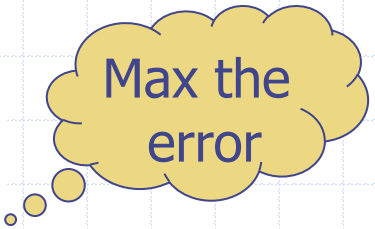
D: address oscillations

P: address errors

[I: address steady state]

Guard ratio

Integral windup



What's next?

- Week 3.6 – **midterm**: yaw control
- Week 3.9 – **final demo**: full cntrl, raw, ...
- Week 3.9 – **report**
  - what you did and **why**
  - results: plots, perf numbers
  - analysis/reflection